Stabilizing unstable periodic orbits in a fast diode resonator using continuous time-delay autosynchronization

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Fast chaotic dynamics in a diode resonator are controlled using a continuous feedback scheme proposed by Pyragas [Phys. Lett. A **181**, 203 (1993)]. The resonator is driven by a 10.3 MHz sinusoidal voltage (corresponding to a drive period under 100 nsec). Period-k orbits, with $k=1$, 2, and 4, have been stabilized by applying a vanishingly small feedback signal that is generated by continuously comparing the state of the resonator with its state one orbital period in the past. We observe that the control is effective even in the presence of a $\sim$24 nsec time lag between the sensing of the system and the application of the feedback that arises from unavoidable propagation delays through the feedback electronics.

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Chaos in a dynamical system can seriously limit its performance in applications where stable behavior is important. Avoiding the domain where chaos occurs may require the design and integration of several systems to cover the full range of desired operating conditions which is not practical in all cases. A more elegant solution is to use a feedback scheme that allows the system to behave in a smooth, stable, and controlled manner even into the chaotic domain. Ideally, the feedback would represent a very slight perturbation of the system so that the desired features of its behavior are not destroyed.

Recently, Ott, Grebogi, and Yorke (OGY, Ref. [1]) pointed out that the unstable periodic orbits (UPO's) of a chaotic system can be exploited to achieve control. Stabilization of an UPO requires minimal perturbation since the orbit already exists (it is determined by the internal dynamics of the uncontrolled system). To demonstrate the feasibility of their proposal, they devised a control algorithm that prescribes how to adjust an accessible control parameter each time the system passes through a chosen Poincaré section (at a point $\mathbf{x}$) so as to guide the system to the desired orbit (corresponding to the fixed point $\mathbf{x}_0$). The strength and sign of the necessary adjustments, which can be thought of as a feedback signal, are determined using linear control theory when $\mathbf{x}$ is in the vicinity of the fixed point [2]. Discrete adjustments proportional to $\mathbf{x}-\mathbf{x}_0$ are made each time the trajectory intersects the Poincaré section until the system is on the UPO, that is, until $\mathbf{x}=\mathbf{x}_0$. Subsequent adjustments are only needed to compensate for noise that drives the system away from the UPO. The OGY scheme has proven to be quite robust to noise and imprecise knowledge of the system as demonstrated by the control of various physical systems [3–9].

Unfortunately, it is difficult to apply the OGY control algorithm to high-speed chaotic systems because the state of the system must be accurately sensed and the feedback signal abruptly changed when the trajectory pierces the Poincaré section. Recently, Pyragas [10] suggested a new algorithm to stabilize UPO's that uses continuous [11,16] rather than abruptly-changing feedback and hence should be better suited for stabilizing high-speed chaotic dynamics. The algorithm synchronizes the system to its state one orbital period in the past by adjusting continuously an accessible control parameter by an amount $\epsilon(t)=\gamma[\xi(t)-\xi(t-\tau_k)]$, where $\xi(t)$ is a measurable system variable and τ_k is the period of the desired UPO [17]. We refer to this process as time-delay autosynchronization (TDAS). When synchronization with the delayed state is successful, the trajectory of the controlled system is precisely on the UPO and the feedback signal is comparable to the noise level in the system. All information required to stabilize the desired orbit is provided by the system in real time, except for the period of the desired UPO and the gain of the feedback loop. We note that TDAS has been used to control the dynamics of an electrical circuit [18] and a laser [19]. However, its high-speed capabilities [20] have not been addressed thoroughly in an experimental system and there are no simple theoretical guidelines to determine whether a system is controllable.

In this paper, we demonstrate that TDAS is effective for stabilizing the chaotic behavior of a high-speed, chaotic electrical circuit: a diode resonator driven by a 10.3 MHz sinusoidal voltage. We emphasize, in particular, that TDAS can stabilize several different UPO's, it automatically tracks changes in the size of the drive voltage (the control parameter), and it is robust with respect to unavoidable time lags associated with elements in the feedback loop. We investigated the diode resonator because it was easy to modify the standard resonator to operate at high speeds, it is well characterized [21], and a variant of the OGY algorithm has been applied to it successfully at low speeds [5]. The resonator consists of a diode (type 1N4007, reverse recovery time $\sim$2 μsec) in series with a 25 μH inductor (series dc resistance of 2.3 Ω) and a 50 Ω resistor which are driven by a sinusoidal voltage $V_d \sin(\omega_d t)$ (frequency $\omega_d/2\pi=10.3$ MHz).

The circuitry for affecting control of the chaotic resonator is shown in Fig. 1. High-speed, low-noise operational amplifiers (Analog Devices, AD811) and low-loss transmission lines were used to create the TDAS error signal and inject it back into the resonator. Homemade printed-circuit boards having well-defined ground planes

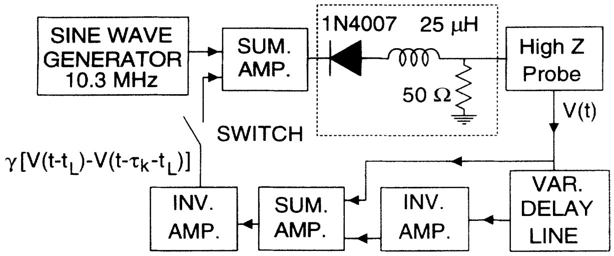

FIG. 1. Block diagram of the experimental setup. The components enclosed within the dashed square makeup the diode resonator which is driven by a sinusoidal voltage. The additional components are used to generate the TDAS error signal and to inject it into the resonator.

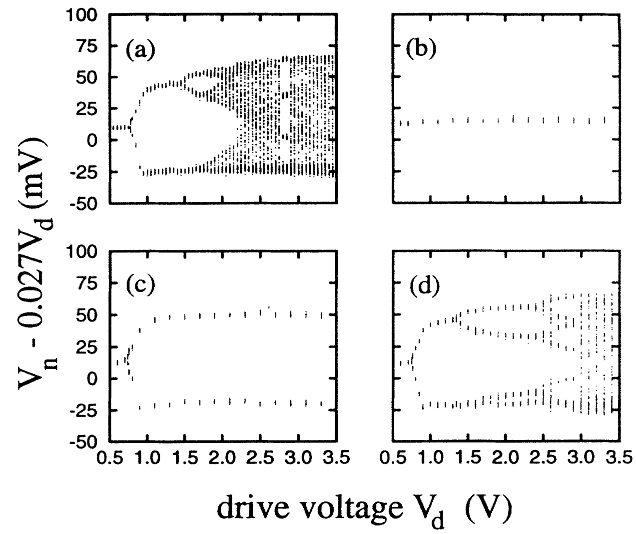

FIG. 2. Bifurcation diagrams of the resonator peak voltage as a function of the drive amplitude. (a) The uncontrolled system. Stabilizing and tracking the (b) period-1 UPO for $\gamma=-3.2$, (c) the period-2 UPO for $\gamma=-2.5$, and (d) the period-4 UPO for $\gamma=-2.5$.

were used for propagating the signals over short distances while commercially available 50 Ω cables (RG 58-U) were used for long-distance propagation to produce the delayed signal. We found that it was convenient to use the voltage drop $V(t)$ across the 50 Ω resistor (proportional to the current flowing through the resonator) as the dynamical variable to characterize the state of the resonator. The voltage $V(t)$ was sensed with a high-impedance buffer to isolate the control circuitry from the resonator and ensure that the observed controlled orbits were indeed orbits of the original system.

The TDAS error signal was derived from $V(t)$ using analog techniques. We directed half of the signal along a short transmission line and into one port of a summing amplifier while the other half was sent along a length of cable, through an inverting amplifier and into the second port of the summing amplifier. The cable length was precisely adjusted so that the signal propagating along it and through the inverting amplifier was delayed by the period τ_k of the desired UPO, where the subscript k signifies a period-k orbit. In our circuit, τ_k is simply given by $2\pi k/\omega_d$. Therefore, we adjusted the time delay to τ_k when the error-generating part of the circuit was disconnected from the rest of the system. We note, however, that the delay time could be adjusted to τ_k in real time while the control circuit was active, which may be required for an autonomous systems where τ_k is not known *a priori*. The amplifier in the delayed-signal path was adjusted to compensate for the loss ($\sim 13\%$ at 10.3 MHz for the period-1 cable, cable length of ~ 19.4 m) in the cable and it inverted the signal so that the output of the summing amplifier was proportional to $[V(t)-V(t-\tau_k)]$. We found that it was necessary to further amplify and invert this signal to establish control of the chaotic diode resonator. Finally, the TDAS feedback signal passed through a switch, was summed with the drive voltage, and injected into the resonator. For closed-loop control, the signal injected into the resonator is given by $V_r(t)=V_d\sin(\omega_d t)+V_\epsilon(t)$, where $V_\epsilon(t)=\gamma[V(t-t_L)-V(t-\tau_k-t_L)]$ denotes the TDAS feedback voltage, γ is the gain of the loop, and t_L is the total time lag introduced by the operational amplifiers. The time lag t_L was ~ 24 nsec, which corresponds approximately to $\frac{1}{4}$ of a drive period. The TDAS scheme appears quite robust against this time lag since control was easily obtained.

Before attempting control, we generated a bifurcation diagram [Fig. 2(a)] to explore the dynamical behavior of the high-speed chaotic resonator. The diagram was made by plotting all the local maxima V_n of $V(t)$ that occurred during a ~ 10 µsec interval for several values of the drive voltage V_d. We performed a simple liner transformation of the data ($V_n \rightarrow V_n - 0.027V_d$, where V_n and V_d are evaluated with the same units) to highlight clearly the dynamics of the resonator. It is seen that the resonator displays a typical period-doubling route to chaos [21], where the first Hopf bifurcation occurs at $V_d \simeq 0.78$ V, and the onset of chaos occurs at $V_d \simeq 1.98$ V.

Control of the resonator was easily initiated by closing the switch in the feedback loop at an arbitrary time and adjusting γ. Successful control was indicated by the observation of a small TDAS error voltage (less than 0.5% of the drive amplitude for all cases, which is comparable to the noise level) and the observation of a stable, periodic form of $V(t)$. We found that there is a finite range of γ (typically an order-of-magnitude) over which control can be maintained for a given drive amplitude. If the gain was too high, the system underwent large-amplitude chaotic behavior and $V_\epsilon(t)$ was comparable to the drive voltage. When the gain was too small, the feedback was insufficient to keep the system on the UPO.

It was possible to stabilize and maintain control of a period-1 UPO over the full range of drive amplitudes using $\gamma=-3.2$. The TDAS error signal automatically tracks [7,19] changes in the UPO as V_d changes because it is self-generating. The tracking behavior of the stabilized period-1 UPO is demonstrated by the bifurcation diagram shown in Fig. 2(b) where it is seen that period-1 behavior is maintained through the first Hopf bifurcation and well into the chaotic domain. Recall that we have transformed the data—the amplitude of the period-1 or-

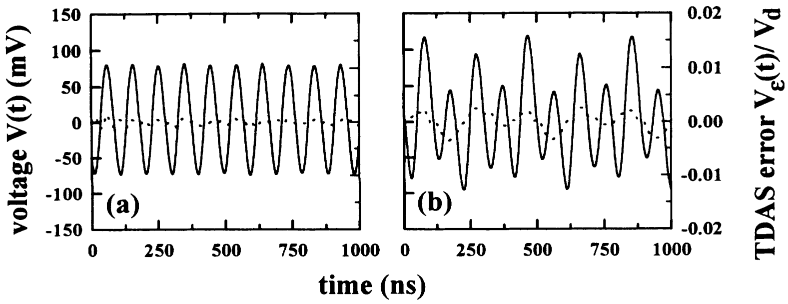

FIG. 3. Temporal evolution of the stabilized (a) period-1 UPO ($\gamma=-3.2$) and (b) period-4 UPO ($\gamma=-2.5$) with the associated TDAS error signal expressed as a fraction of the drive amplitude for $V_d=2.4$ V.

bit increases for increasing drive amplitudes.

We have observed the transient behavior following initiation of period-1 control. For drive amplitudes that placed the system in the chaotic regime, it took anywhere from ~ 0.5 μsec (best case) to ~ 100 μsec (worst case) to stabilize the period-1 orbit. During the longer transients, the system often displayed nearly stable period-2 or -4 behavior as it converged to the period-1 orbit.

We have also stabilized period-2 and -4 UPO's using appropriate lengths of the signal-delay cable. Figure 2(c) shows the stabilization of a period-2 orbit using $\gamma=-2.5$ with $\tau_2=4\pi/\omega_d$. For $V_d\geq 0.78$ V, the TDAS scheme maintains control over the period-2 orbit throughout the chaotic regime. The domain of control for the period-4 orbit did not extend over the entire chaotic regime as shown in Fig. 2(d), where $\gamma=-1.4$ with $\tau_4=8\pi/\omega_d$. It is seen that control is maintained only over the range $1.5\leq V_d\leq 2.5$ V. For higher drive amplitudes, no value of γ would stabilize the period-4 orbit.

To demonstrate that the TDAS signal is a small fraction of the drive amplitude when the control is effective, we simultaneously plot the temporal evolution of the error signal along with $V(t)$ for $V_d=2.4$ in the chaotic regime. Figure 3(a) shows the stabilized period-1 orbit and its associated TDAS error signal for $\gamma=-3.2$. The peak error signal is much less than 0.2% of the drive amplitude and is not distinguishable from the noise. The situation is slightly different for the error signal associated with the stabilized period-4 orbit shown in Fig. 3(b) which is for the case $\gamma=-1.4$. The error signal is just distinguishable from the noise but is no larger than 0.4% of the drive amplitude. We believe that the increase in the amplitude of the error signal is due to frequency-dependent loss (distortion) in the delay line which results in an imperfect TDAS signal (we compensate only for a frequency-independent loss).

We stress that the TDAS scheme stabilizes an UPO of the chaotic system rather than creating new periodic orbits that are not directly related to the system. We demonstrate this property by superimposing the first return map of the controlled system over the map of the chaotic system. If the stabilized orbit is an UPO of the system, its position will be located precisely on the map of the uncontrolled system. The maps shown in Fig. 4 were generated by plotting the voltage V_n of the nth peak versus the voltage V_{n+1} of the preceding peak. Figures 4(a)–4(c) are for $V_d=2.4$ V. Figure 4(a) shows that the stabilized period-1 orbit is indeed an UPO of the system. It corresponds precisely to the intersection of the uncontrolled return map with the line $V_n=V_{n+1}$. The experimental conditions are the same used to generate Fig. 3(a). Similar results are shown for the period-2 (-4) orbit in Fig. 4(b) [4(c)] for the same conditions used to generate Figs. 2(c) [3(b)].

We have also investigated the effect of TDAS on the system when control of the period-4 orbit is attempted for high drive amplitudes ($V_d=2.6$ V). Figure 4(d) shows the return map of the system with and without TDAS feedback where it is seen that the feedback is ineffective at controlling the period-4 orbit. In this case, the size of the error signal was several percent of the drive voltage. Hence, the smallness of the TDAS error signal indicates simply whether the observed motion is a UPO, thus avoiding the complication of generating a return map. We note that periodic orbits that are not UPO's of the system could be obtained using the continuous feedback

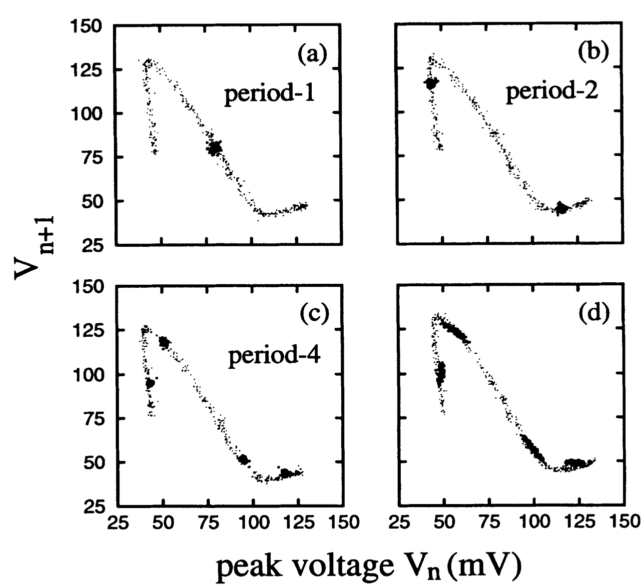

FIG. 4. First return maps demonstrating that the controlled dynamics of the resonator are UPO's of the system. In each plot, the map of the controlled trajectory (dark concentration of points) is superimposed on top of the map of the uncontrolled (chaotic) system (lighter points). For (a)–(c), $V_d=2.4$ V. Controlled (a) period-1 UPO ($\gamma=-3.2$), (b) period-2 UPO ($\gamma=-2.5$), and (c) period-4 UPO ($\gamma=-2.5$). (d) Unsuccessful control of the period-4 orbit for $V_d=2.6$ V and the same parameters as in (c).

scheme. However, the TDAS error signal was large in these cases.

Finally, we comment on how an all-optical implementation of TDAS might be used to control chaotic diode lasers, which would be nearly impossible using traditional techniques because the chaotic fluctuations in the intensity of the laser (often caused by weak stray reflections of light back into the laser) occur on a nanosecond or subnanosecond time scale [22]. In the all-optical implementation, a small fraction of the light generated by the laser would be used to create the TDAS error signal. The necessary time delay and subtraction would be accomplished using the well-known properties of an optical interferometer (a Michelson or Fabry-Pérot interferometer, for example) and the gain of the feedback loop would be adjusted by varying the fraction of the light sampled from the laser beam. Note that TDAS would require only a small fraction of the light generated by the laser because the laser resonator amplifies the effects of any field injected into it due to its large quality factor, or finesse.

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